1. Is it possible that an event is independent of itself? If so, when?
2. Is it always true that if A and B are independent events, then Ac and Bc are independent events? Show that it is, or give a counterexample.

Answer:

1. No, it is not possible for an event to be independent of itself. Independence between two events means that the occurrence of one event does not affect the probability of the other event occurring. If we consider a single event, it either occurs or does not occur, and its occurrence is necessarily related to its non-occurrence. Therefore, there is no way for an event to be independent of itself.
2. It is not always true that if A and B are independent events, then Ac and Bc are independent events. To see why, consider the following counterexample:

Let A and B be independent events such that P(A) = P(B) = 1/2. Then, P(Ac) = P(Bc) = 1/2 as well. Now, consider the event (A ∩ Bc). We have:

P(A ∩ Bc) = P(A)P(Bc) = (1/2)(1/2) = 1/4

However, we also have:

P(A)P(Bc) = (1/2)(1/2) = 1/4 = P(A)P(Bc)

Thus, we see that (A ∩ Bc) and (Ac ∩ B) are not independent events, even though A and B are independent events. Therefore, it is not always true that if A and B are independent events, then Ac and Bc are independent events.